

## An Investigation of NRD waveguide Grating

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## ABSTRACT

The filter characteristic of NRD waveguide grating is investigated rigorously by combining network approach with mode matching theory. Numerical examples are shown and compared with experiment results.

## INTRODUCTION

NRD waveguide has been widely applied to various components in millimeter-wave techniques because of its superior characteristics such as low loss, suppressing radiation, etc. The NRD waveguide with periodic corrugation can also be widely used in millimeter-wave regions by employing the Bragg reflection or the leaky-wave phenomena. For example, it can be used in DBR Gunn oscillator to stabilize the oscillation frequency. Some dielectric grating with finite length have been analysed by some authors (1), (2), (3), (4). In (1) and (2), the method applied is too complex to practice, while in (3), the method is too simple and results in some errors. In this paper, we present an effective approach to analyse a periodic structure on NRD waveguide by combining network approach with mode matching theory. Our approach is simpler than that in (1) and (2), but can still ensure higher precisions.

## THEORETICAL ANALYSIS

The main procedure of our approach to analyse a periodic structure on NRD waveguide shown in Fig.1 is as follows:

(1) First, we consider that the structure consists of  $N$  basic cells and each cell comprises three NRD waveguide sections with two step discontinuities, as shown in Fig.2. Only the dominant mode of NRD waveguide can be transmitted along the guide.

(2) Secondly, we find out the network representations of each cell by combining network approach with mode matching.

(3) Finally, we can give the network representing the whole periodic structure

by cascading the subnetwork representing each cell.

The key of the approach is to find out the network of each cell.

Modes of NRD guide are hybrid in nature, having both electric and magnetic components in the longitudinal direction. They are usually referred to as longitudinal-section magnetic (LSM) and longitudinal-section electric (LSE) modes. The operating mode of NRD guide is the lowest LSM mode, i.e. LSM<sub>11</sub> even mode. The transverse fields of the desired modes may be written as follows:

$$\begin{aligned}\vec{E}_t(x, y, z) &= \sum_i V_i(z) \vec{e}_i(x, y) \\ \vec{H}_t(x, y, z) &= \sum_i I_i(z) \vec{h}_i(x, y)\end{aligned}\quad (1)$$

where  $V_i$ ,  $I_i$ ,  $\vec{e}_i$  and  $\vec{h}_i$  satisfy the following equations, respectively

$$\frac{dV_i}{dz} = -jKz_i Z_i I_i, \quad \frac{dI_i}{dz} = -jKz_i Y_i V_i \quad (2)$$

$$(\nabla_t^2 + K_{zi}^2) \vec{e}_i = 0, \quad (\nabla_t^2 + K_{zi}^2) \vec{h}_i = 0 \quad (3)$$

where  $K_{zi}^2 = K^2 - K_{zi}^2 = K_{xi}^2 + K_{yi}^2$

$$\vec{e}_i = \vec{x}e_{xi} + \vec{y}e_{yi}$$

$$\vec{h}_i = \vec{x}h_{xi} + \vec{y}h_{yi}$$

The mode function  $\vec{e}_i$  and  $\vec{h}_i$  possess following orthogonality properties

$$\oint (\vec{e}_i \times \vec{h}_j) \cdot \vec{z}_0 dx dy = \delta_{ij} \quad (4)$$

The relations between  $\vec{e}_i$  and  $\vec{h}_i$  are

$$\begin{aligned}Kz_i Z_i \vec{e}_i &= \omega \mu (\vec{I}_t + \frac{\nabla_t \nabla_t}{K^2}) \cdot (\vec{h}_i \times \vec{z}_0) \\ Kz_i Y_i \vec{h}_i &= \omega \mu (\vec{I}_t + \frac{\nabla_t \nabla_t}{K^2}) \cdot (\vec{z}_0 \times \vec{e}_i)\end{aligned}\quad (5)$$

Where  $\vec{I}_t = \vec{x}\vec{x} + \vec{y}\vec{y}$

(2) and (3) are the transmission line equations and the vector eigenvalue problem, respectively.

For the LSE modes, where  $e'_{yi} = 0$ , so that

$$e'_{xi} = h'_{yi} \quad (6)$$

For the LSM mode, where  $h'_{yi} = 0$ , so that

$$h_{xi} = -\epsilon_r(y) e_{yi} \quad (7)$$

The following scalar orthogonality conditions may be written as a consequence of (4), (6) and (7)

$$\begin{aligned} \iint e_{xi} e_{xj}^* dx dy &= \delta_{ij} \\ \iint e_{yi} \epsilon_r(y) e_{yj}^* dx dy &= \delta_{ij} \\ \iint h_{xi} h_{xj}^* dx dy &= \delta_{ij} \\ \iint h_{xi} 1/\epsilon_r(y) h_{xj}^* dx dy &= \delta_{ij} \end{aligned}$$

From above relations we can find out transverse components of total fields in NRD guide.

When the dominant mode (LSM11 even mode) of NRD guide injects into a step with an arbitrary incident angle, it will arise many higher order modes, as well as continuous spectrum modes. But because of the symmetry of NRD guide and no discontinuity in x direction, the parasitic disperse modes at the step are only LSM1n even modes and LSM1n odd modes. The desired transverse fields at both sides of step are

$$\begin{aligned} \text{Guide I} \quad E_x &= \sum_i V_i e_{xi} + \sum_i V_i' e_{xi}' \\ E_y &= \sum_i V_i e_{yi} \\ H_x &= \sum_i I_i h_{xi} + \sum_i I_i' h_{xi}' \\ H_y &= \sum_i I_i h_{yi} \\ \text{Guide II} \quad E_x &= \sum_i \tilde{V}_i \tilde{e}_{xi} + \sum_i \tilde{V}_i' \tilde{e}_{xi}' \\ E_y &= \sum_i \tilde{V}_i \tilde{e}_{yi} \\ H_x &= \sum_i \tilde{I}_i \tilde{h}_{xi} + \sum_i \tilde{I}_i' \tilde{h}_{xi}' \\ H_y &= \sum_i \tilde{I}_i \tilde{h}_{yi} \end{aligned}$$

Here the continuous spectrum modes have been omitted.

Matching the fields at step T1 and multiplying the equations with  $e_{mx}$ ,  $e_{my}$ ,  $r(y)$ ,  $h_{mx}/\epsilon_r(y)$  and  $h_{my}$ , respectively, and making integral around cross-section of the NRD guide, we can get following equations by employing the orthogonality and normalized conditions

$$V' + R''V'' = Q' \tilde{V}' + S'' \tilde{V}'' \quad (8)$$

$$V'' = P' \tilde{V}' \quad (9)$$

$$R' I' + I'' = S' \tilde{I}' + Q'' \tilde{I}'' \quad (10)$$

$$I' = P' \tilde{I}' \quad (11)$$

where  $I'$ ,  $\tilde{I}'$ ,  $V''$  and  $\tilde{V}''$  are column vectors of the modal currents and voltages of LSM mode, while  $I'$ ,  $\tilde{I}'$ ,  $V'$  and  $\tilde{V}'$  represent LSE modal currents and voltage.  $R'$ ,  $R''$ ,

$Q'$ ,  $Q''$ ,  $S'$ ,  $S''$ ,  $P'$  and  $P''$  are matrices showing the coupling relations between modes. The elements of the matrixes are

$$\begin{aligned} R_{mn} &= \langle e_{mx}' | e_{nx}'' \rangle & Q_{mn} &= \langle e_{mx}' | \tilde{e}_{nx}'' \rangle \\ S_{mn} &= \langle e_{mx}' | \tilde{e}_{ny}'' \rangle & P_{mn} &= \langle e_{my}' | \epsilon_r(y) | \tilde{e}_{ny}'' \rangle \\ R_{mn} &= \langle h_{mx}'' | 1/\epsilon_r(y) | h_{nx}' \rangle \\ S_{mn} &= \langle h_{mx}'' | 1/\epsilon_r(y) | \tilde{h}_{nx}' \rangle \\ Q_{mn} &= \langle h_{mx}'' | 1/\epsilon_r(y) | \tilde{h}_{nx}' \rangle \\ P_{mn} &= \langle h_{my}'' | \tilde{h}_{ny}' \rangle \end{aligned}$$

The step T2 can be analysed similarly with the step T1, it is omitted here. Using above relations, we can readily get the network representation for a step of NRD guide with generalized ABCD matrix.

In addition, we consider a NRD section as multiple transmission lines and can also get its network representation with generalized ABCD matrix. Then the network of a basic cell can be got by cascading the networks for steps and networks for NRD guide sections as shown in Fig.3 (a). Finally, the equivalent network of the periodic structure shown in Fig.1 can be work out by cascading the subnetworks presenting the basic cells shown in Fig.3 (b).

The network shown in Fig.4 is a multiple ports network for the whole periodic structure, we can easily reduce it to a two ports network by terminating the ports representing higher order modes with their characteristic impedances.

#### NUMERICAL AND EXPERIMENT RESULTS

A series of periodic structures have been computed. The results of computing are shown in Fig.5 (a). In order to examine the theoretical analysis, one of the examples has been made and measured. The result is shown in Fig.5 (b). The results show that the agreement between theoretical analysis and experiment results is good.

#### CONCLUSION

This paper presents the equivalent network and the filter characteristics of a periodic NRD guide structure. This structure can be used in millimeter wave dielectric waveguide integrated oscillator (DBR oscillator) as a frequency stabilizer. The approach to analyze the periodic NRD guide structure is given for the first time and is a effective method. The numerical and experiment results agree well.

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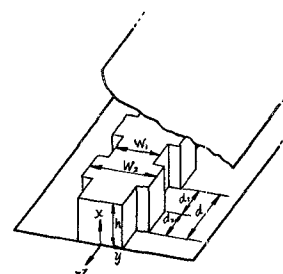


Fig.1 NRD guide periodic structure

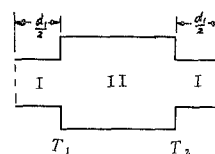


Fig.2 A basic cell of the periodic structure

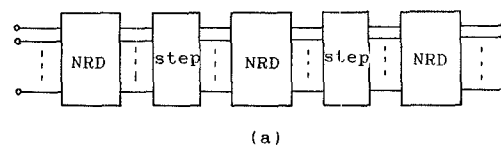


Fig.3 (a) The cascade of networks for steps and for NRD sections

(b) The network for a basic cell

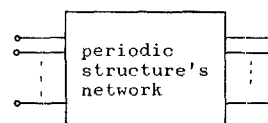
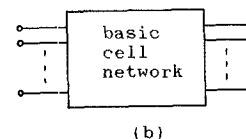
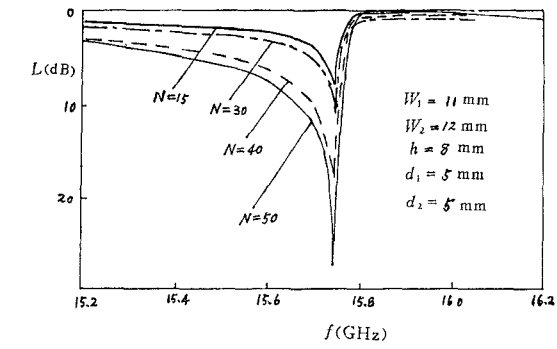
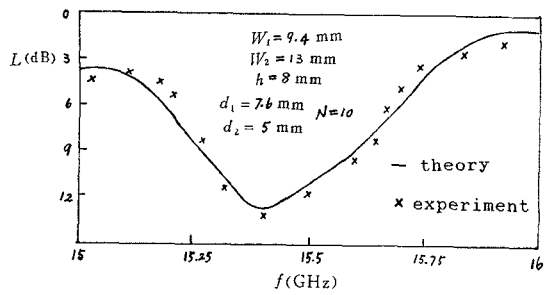


Fig.4 The network for the whole periodic structure



(a)



(b)

Fig.5 (a) the examples of computed attenuations for periodic structures with a finite length  
(b) the theoretical and measured attenuations for a periodic structure with a finite length